Lesson 2. Vectors

1 In this lesson...

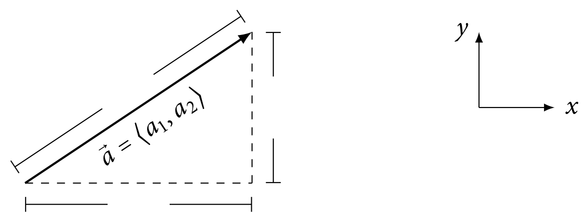
- Vectors in $\mathbb{R}^2$ and $\mathbb{R}^3$
- Standard basis vectors and unit vectors
- Problems with forces

2 What is a vector?

- A **vector** is an object that has both [ ] and [ ]
- Notation: **a** or $\vec{a}$
- A vector $\vec{a}$ can be represented by an ordered list of numbers:

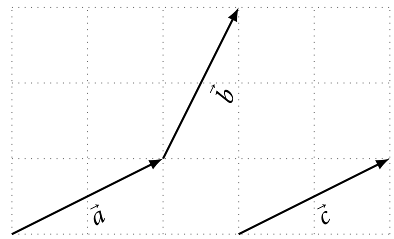
$$\vec{a} = \langle a_1, a_2 \rangle \quad (\text{in } \mathbb{R}^2) \qquad\qquad \vec{a} = \langle a_1, a_2, a_3 \rangle \quad (\text{in } \mathbb{R}^3)$$

- These numbers (e.g. a_1, a_2, a_3) are known as **components** of $\vec{a}$
- For now, let's stick to $\mathbb{R}^2$
  - Much of what we'll see generalizes to $\mathbb{R}^3$
- Graphically:



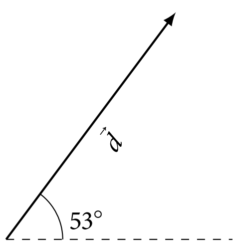
- The **magnitude** or **length** of vector $\vec{a} = \langle a_1, a_2 \rangle$ is [ ]
- Two vectors are **equivalent** if they have the same magnitude and direction – position does not matter
- A special vector – the **zero vector** $\vec{0} = \langle 0, 0 \rangle$
  - $\vec{0}$ is the only vector with no specific direction

Example 1. Consider the vectors below.



a. Give $\vec{a}$, $\vec{b}$, $\vec{c}$ as an ordered list of numbers.
b. Find $|\vec{c}|$.
c. Are any of these vectors equivalent? Which ones?

Example 2. Consider $\vec{d}$ below. We are given that $|\vec{d}| = 5$. Give $\vec{d}$ as an ordered list of numbers.



3 Scalar multiplication

- Let c be a scalar, $\vec{a}$ be a vector $\Rightarrow$ $c\vec{a}$ is another vector
- $c\langle a_1, a_2\rangle =$

Example 3. Consider $\vec{a} = \langle 2, 1\rangle$ from the previous example.

a. $3\vec{a} =$

b. $-2\vec{a} =$

c. Draw $\vec{a}$, $3\vec{a}$ and $-2\vec{a}$ below.

- If $c > 0$, then $c\vec{a}$ is a vector in the same direction as $\vec{a}$ and $|c|$ times the length of $\vec{a}$
- If $c < 0$, then $c\vec{a}$ is a vector in the opposite direction as $\vec{a}$ and $|c|$ times the length of $\vec{a}$
- If $c = 0$, then $c\vec{a} = \vec{0}$

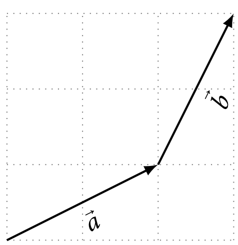
4 Adding and subtracting vectors

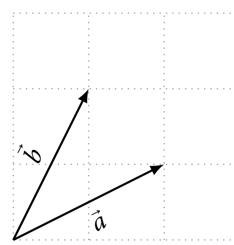
- Let $\vec{a}, \vec{b}$ be vectors $\Rightarrow$ $\vec{a} + \vec{b}$ is another vector
- $\langle a_1, a_2 \rangle + \langle b_1, b_2 \rangle =$

Example 4. Consider $\vec{a} = \langle 2, 1 \rangle$ and $\vec{b} = \langle 1, 2 \rangle$ from the previous example.

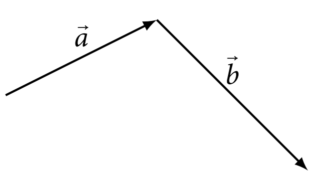
a. $\vec{a} + \vec{b} =$

b. Draw $\vec{a} + \vec{b}$ below:

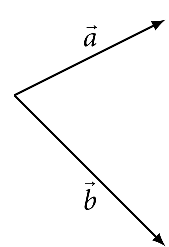




- **Triangle law** for adding vectors:



- **Parallelogram law** for adding vectors:



- $\langle a_1, a_2 \rangle - \langle b_1, b_2 \rangle =$

5 Generalizations

- All of the above generalizes naturally to $\mathbb{R}^3$:

$$|\langle a_1, a_2, a_3\rangle| = \sqrt{a_1^2 + a_2^2 + a_3^2} \qquad \langle a_1, a_2, a_3\rangle + \langle b_1, b_2, b_3\rangle = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3\rangle$$

$$c\langle a_1, a_2, a_3\rangle = \langle ca_1, ca_2, ca_3\rangle \qquad \langle a_1, a_2, a_3\rangle - \langle b_1, b_2, b_3\rangle = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3\rangle$$

- Algebraically, vectors behave a lot like scalars, e.g.

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \qquad c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b} \qquad (c + d)\vec{a} = c\vec{a} + d\vec{a}$$

- See p. 802 of Stewart for a fuller list

6 Standard basis vectors and unit vectors

- **Standard basis vectors** in $\mathbb{R}^3$:

$\vec{i} =$ $\qquad$ $\vec{j} =$ $\qquad$ $\vec{k} =$

- We can write any vector as the sum of scalar multiples of standard basis vectors:

- A **unit vector** is a vector with length 1
  - For example, $\vec{i}, \vec{j}, \vec{k}$ are all unit vectors

- The unit vector that has the same direction as $\vec{a}$ (assuming $\vec{a} \neq \vec{0}$) is

Example 5. Let $\vec{a} = 4\vec{i} - \vec{j} + 2\vec{k}$ and $\vec{b} = \vec{i} + 2\vec{k}$.

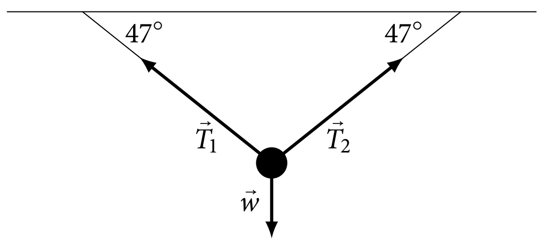
a. Write $\vec{a} - 2\vec{b}$ in terms of $\vec{i}, \vec{j}, \vec{k}$.
b. Find a unit vector in the direction of $\vec{a} - 2\vec{b}$.

- Note: all of this applies to vectors in $\mathbb{R}^2$ in a similar way

7 Problems with forces

- Some physics:
  - **Force** has magnitude and direction, and so it can be represented by a vector
  - Force is measured in pounds (lbs) or newtons (N)
  - If several forces are acting on an object, the **resultant force** experienced by the object is the sum of these forces

Example 6. A weight $\vec{w}$ counterbalances the tensions (forces) in two wires as shown below:



The tensions $\vec{T}_1$ and $\vec{T}_2$ both have a magnitude of 20lb. Find the magnitude of the weight $\vec{w}$.

- Note: if an object has a mass of m kg, then it has a weight of mg N, where $g = 9.8$